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Fractals in Earthquakes

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Phil. Trans. R. Soc. Lond. A 1994 **348**, 449-457
doi: 10.1098/rsta.1994.0104

Fractals in earthquakes†

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Earthquake prediction is one of the most important probrems for countries on a plate margin with high seismic activity. The earthquake is a typical example of a common phenomena which has several kinds of fractal features. We introduce the 'stick-slip model', which can explain the fractal features of seismic phenomena to the earthquakes in Japan and discuss about the predictiblity of the destructive earthquakes.

1. Introduction

An earthquake is a fracture process of the crust. It both microscale and macroscale features, for example, the phase transition of rock-forming minerals, microcracks of rocks, slip at the active fault or plate boundary.

There are several kinds of scale invariance or fractal features in the earthquake phenomena which are given in a power law form, for example, the magnitude–frequency distribution, the spatial distribution of hypocentres, the frequency of aftershock occurrences and so on. We discuss these interesting features in the next section.

2. Fractal in the earthquake phenomena

The magnitude–frequency distribution of earthquakes is described by

$$\log N(>M) = -bM + \text{const.}, \tag{2.1}$$

where M is the magnitude of the earthquake and $b \approx 1.0$ (Kanamori & Anderson 1975). In this formula $N(>M)$ denotes the number of earthquakes having magnitude larger than M. This is the famous Gutenberg–Richter's law.

The magnitude M is proportional to the logarithm of the energy E of the seismic event, just as $M \propto \frac{2}{3}\log E$, so we can get the following formula for the distribution of energy from (2.1):

$$\log N(>M) = -\tfrac{2}{3}b\log E + \text{const.} \tag{2.2}$$

Since the magnitude is proportional to the area of the fracture zone (Utsu 1969),

$$\log S = M - 3.7, \tag{2.3}$$

† This paper was produced from the author's disk by using the TEX typesetting system.

Phil. Trans. R. Soc. Lond. A (1994) **348**, 449–457
Printed in Great Britain

the Gutenberg–Richter's law can also be written as

$$\log N(>S) = -b \log S + \text{const.}, \tag{2.4}$$

where S is the area of the earthquake's fracture zone (Kanamori & Anderson 1975).

The size of the earthquake can be estimated from the length of the fault. As the fracture-area–fault-length relationship is described as $\log S \propto 2 \log L$, we can get the following formula from (2.4):

$$\log N(>L) = -2b \log L + \text{const.} \tag{2.5}$$

Thus the size–frequency distribution of earthquakes is given in the power law form.

The spatial distribution of hypocentres of earthquakes is obviously heterogeneous. Its fractal nature was first demonstrated by Kagan & Knopoff (1980) and Kagan (1981), who studied the spatial distribution of earthquake hypocentres and epicentres, using world wide as well as local catalogues. They found that the correlation functions, which describe the number of events per unit volume or area at a distance R from any earthquake, can be described as follows:

$$\log N(R) \propto a \log R \tag{2.6}$$

over a certain range of the distance R. The exponent a corresponds to the fractal dimension D as in $D = d - a$, where d is the spatial dimension ($d = 2$ for an epicentre distribution and $d = 3$ for a hypocentre distribution).

The number of events per unit volume occurring in distance R from any earthquake is given by

$$\log N(<R) = D \log R + \text{const.} \tag{2.7}$$

D in two dimensions ranges from 1.0 for the world wide catalogue, 1.1 for California and 1.3 for Japan. Sadovskiy *et al.* (1984) also found a self-similar feature of the earthquake distribution and obtained a value of fractal dimension D as 1.4 for the world wide catalogue and 1.6 for a local Soviet catalogue. Yoshida & Mikami (1986) reported $D = 2.7$ for the aftershock hypocentre distribution in Nagano Japan, which corresponded to $D = 1.7$ for the epicentre distribution.

As the spatial distribution of hypocentres is confirmed as fractal, we can suppose that earthquakes have some kind of interaction. In particular, a mainshock and its aftershocks must have strong interaction.

The interaction of earthquakes is shown by the power law decrease of aftershocks. The decrease of aftershocks is described as follows:

$$P(t) = K/(t + c), \tag{2.8}$$

where $P(t)$ represents the probability of aftershock occurrences in time t after the main shock, K is the value proportional to the seismic energy and c is a constant introduced to avoid the divergence at $t = 0$. This formula shows that the decrease of aftershocks is proportional to $1/t$ (Omori's law).

As already shown, aftershocks have a scale invariance on time. But mainshocks have been considered as random phenomena having no interactions. But if we consider mainshocks and aftershocks simultaneously, we obtain the same power law relationship about the time sequence that is shown in the case without mainshocks. Recently this topic is attracting much attention. Ogata (1988) showed

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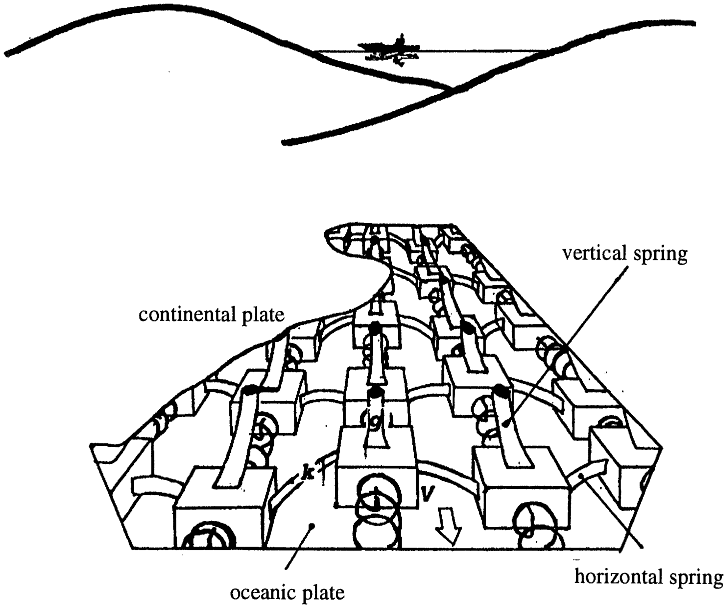

Figure 1. The schematic representation of the two-dimensional stick-slip model. The oceanic plate moves with a constant velocity V and oscillators make a square lattice on the surface of the oceanic plate. (After Matsuzaki & Takayasu.)

that the correlation function of the occurrence of earthquakes follows the power law as

$$\log C(t') = -b \log t' + \text{const.}, \tag{2.9}$$

where $C(t')$ is the correlation function for time interval t'.

3. Two-dimensional stick-slip model

The two-dimensional stick-slip model of Matsuzaki & Takayasu (1991) consists of oscillators arranged on a square lattice. The oscillators are on the frictional surface moving with a constant velocity. Each oscillator is interconnected with the four surrounding oscillators by horizontal springs of rigidity k and also connected to the rigid support by a vertical spring of rigidity g (figure 1). While the sum of the forces from the horizontal springs and vertical spring f at every site is smaller than the frictional force fc the oscillators move with a constant velocity V and their spring forces increase by Vg. When the sum of the spring forces $f(i,j)$ at the (i,j) site exceeds the threshold frictional force $fc(i,j)$, the oscillator slips back to the position at which the total spring force is equal to 0, and the forces that were acting on it are distributed to the surrounding oscillators except the ones that have already slipped at the same time step.

To avoid unnecessary complication we assume that the spring constants k and g are identical for all oscillators. Normalizing the variables by the threshold, we have the following rule:

$$\mathrm{d}F(i)/\mathrm{d}t = v(i,j); \quad \text{if} \quad F(i,j) > 1.0,$$

then $F(i,j)=0$, $F(i{+}1,j{+}1)=F(i{+}1){+}d$, $F(i{-}1,j{-}1)=F(i{-}1,j{-}1){+}d$, where $F(i,j)=f(i,j)/fc(i,j)$, $v(i,j)=Vg/fc(i,j)$, $d=k/(g+4k)$.

Because the thresholds are randomly distributed, the normalized time derivatives of forces, $v(i,j)$, distribute randomly. A slip of the oscillator at (i,j) creates an increase of the force on the surrounding oscillators by d. If $F(i+1,j+1)$ or $F(i-1,j-1)$ becomes larger than 1.0 by this effect, the slip extends to the neighbouring oscillators except the one which has already slipped. When all force values are less than 1.0, the slip propagation stops. A cluster is defined as a group of oscillators which slipped at one time step through this process.

In this model, the cluster size gives the fracture area of an earthquake and the force drop is proportional to the moment of an earthquake. This model has a parameter d, which is related to the ratio of vertical spring to the horizontal spring constant. For small d, the events tend to be localized and the system belongs to a random phase. Conversely, for large d, the force of the slipped oscillator is nearly conserved, so the events tend to extend over a large region and the system belongs to the coherent phase.

These two phases are separated at a critical value of d. At the critical point, the model is consistent with the empirical laws of earthquakes such as Gutenberg–Richter's law, the spatial distribution of hypocentres, and the correlation function of the earthquake occurrences.

Although the correlation function of the time sequence of this model is consistent with that of real seismic phenomenon, it is known that aftershocks are much fewer than real cases.

To improve the two-dimensional stick-slip model so that aftershocks may occur, we introduce a hypothesis of aftershock occurrences as follows. A main shock causes the strain decrease and the crust of the focus region may melt partially. Aftershock may be viewed as the relaxation process of the state of the crust from the partial melting state, in which little strain energy can be held, to the solid state, in which much strain energy can be held. This assumption is based on the entropy relaxation. The low viscosity area corresponds to the perturbed area of the cellular automaton model by Ito & Matsuzaki (1991).

To realize this scenario we modify the evolution rule as

$$\mathrm{d}F(i)/\mathrm{d}t = v(i,j); \quad \text{if} \quad F(i,j) > 1.0,$$

$$\text{then} \quad F(i,j)=0, \quad F(i+1,j+1)=F(i+1)+d,$$

$$F(i-1,j-1)=F(i-1,j-1)+d, \quad Fc(i,j)=0.01$$

in a certain period tr. For example, $tr = 100/\langle v(i,j)\rangle$ (angle brackets denote an average over (i,j)) in the following simulation, which is long enough to change the behaviour of the system from the original two-dimensional stick-slip model. When the period tr is short, the behaviour of the system is almost equivalent to the original model. This case corresponds to the situation that the crust of the fault region rarely melts. When the period tr is long, the behaviour of the system is very different from that of the original two-dimensional stick-slip model. This case corresponds to the situation that aftershocks occur for a long period because the state of the fault region does not return to the steady state. The maximum period in real seismic phenomena is estimated as 10 000 days (Ogata 1987).

Figure 2 shows a pattern of the aftershock occurrences after a big event of

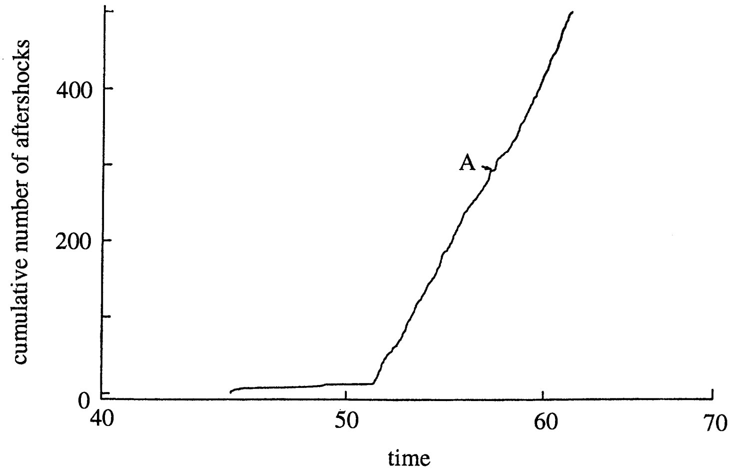

Figure 2. Cumulative number of aftershocks after a large event, whose cluster size is comparable to the system size. Note that the time axis is a log scale.

the reduced threshold model. When a main shock occurs, the slope of this curve increases because many aftershocks occur in a small period. This is caused by the decrease of thresholds of the oscillators. When the thresholds return to the original values, the interval of two nearest events becomes longer and then the next mainshock occurs. The biggest aftershock occurs at the point of A of this curve, after a comparatively long interval.

4. One-dimensional stick-slip model

The one-dimensional version of the stick-slip model described by Matsuzaki & Takayasu (1991) is applied for active fault planes facing two crustal blocks. A frictional box on a constantly moving crustal block is connected to the adjacent crustal block by a leaf spring constant which is g, and also connected to the two nearest neighbour boxes by coil springs whose constants are k.

The evolutional rule of this system which normalized by the thresholds is described as follows, just like the two-dimensional model:

$$\mathrm{d}F(i)/\mathrm{d}t = v(i), \quad \text{if} \quad F(i) > 1.0,$$

$$\text{then} \quad F(i) = 0, \quad F(i+1) = F(i+1) + d, \quad F(i-1) = F(i-1) + d,$$

where $F(i) = f(i)/fc(i)$, $v(i) = Vg/fc(i)$.

When we assume that spring constants g and k are identical for each spring, the temporal behaviour of this system is determined by a parameter, $d = k/(g+2k)$, which is related to the ratio of leaf and coil springs, just like the two-dimensional model. For small d, a large part of the force drop cannot distribute to the surrounding oscillators and events tend to be localized. This feature corresponds to the creaping of the fault. Conversely, for large d, the force drop of the slipped oscillator is distributed to the connecting oscillators, and eventually extends over a large region.

In this study, we changed the value of model parameter d in the range from 0.2 to 0.3. A model with smaller value results in a number of small events, while

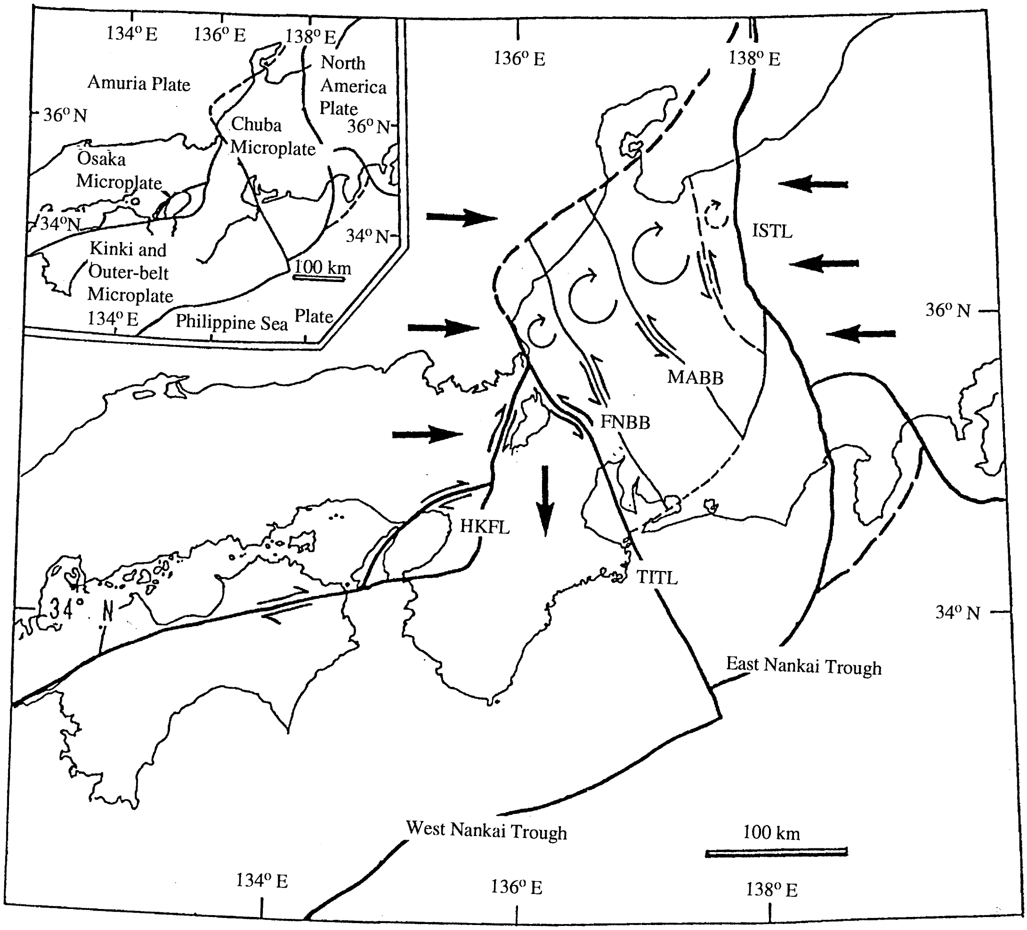

Figure 3. The block rotation model of Southwest Japan. This area is under the pressure due to the plate motion, and the rotation of the three or four blocks makes the coupling of the seismic phenomena. (After Kanaori *et al.* 1993.)

a model with larger d value results in only one great event, comparable to the system size, confirming the above statement. For $d = 0.23$, the results exibit the results exhibit a few large events similar to those obtained by the historical earthquake analysis. It is interesting to note that the two-dimensional stick-slip model with a value of $d = 0.23$ generates a number of events whose size distribution is consistent with Gutenberg–Richter's law (Matsuzaki & Takayasu 1991). Although the one-dimensional stick-slip model studied here couldn't exhibit the fractal features of the earthquake phenomena, it provides rich information for seismic couplings between adjacent fault segments (Hung & Turcotte 1990).

5. The caterpillar model

Kanaori *et al.* (1991*a*, *b*, 1992, 1993) investigated the space-time patterns of destructive earthquake occurrences in Central Japan and proposed a new model of seismic cycles. In this model, block boundaries or tectonic lines are defined as

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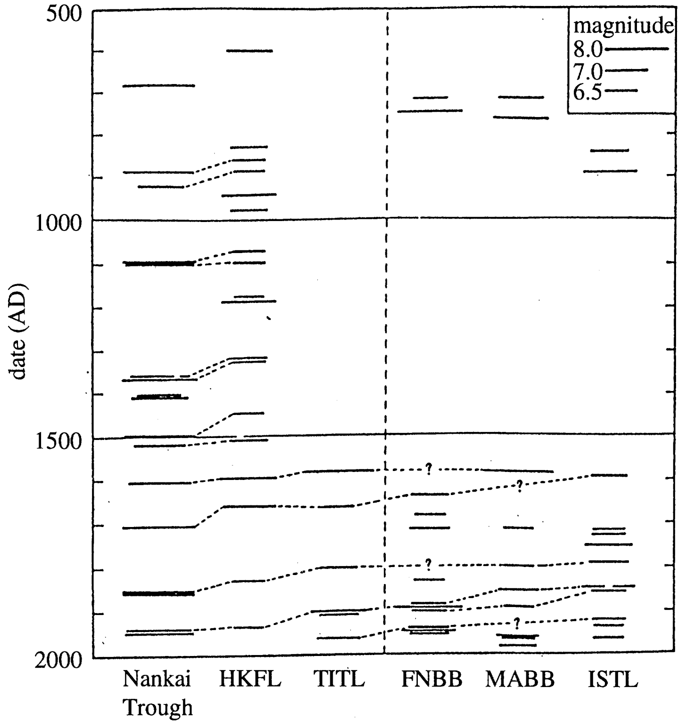

Figure 4. The space-time patterns of the Southwest Japan estimated from historical earthquakes. Each event is relocated along the block boundaries and the length of each line corresponds to the magnitude of each event. We can find the seismic coupling in this area. (After Kanaori *et al.* 1993.)

linking major active faults distributed in the area (figure 3). In this figure, we can find a sliding block and some rotating blocks. If we assume this block rotation model, we can see the seismic coupling between these block boundaries.

The space-time patterns of destructive earthquakes with a magnitude of 6.4 or greater have been analysed by relocating the postulated epicentres to the corresponding block boundaries or tectonic lines. This study revealed that the destructive earthquakes in that area have clear periodicities of active and quiet intervals with a period of about 1000 years and the duration of active periods is several hundred years. We can also find the seismic coupling between West Nankai Trough and other block boundaries, which is the empirical feature of seismic events along West Nankai Trough occuring 30 years after the earthquake HKFL (Hanaore–Kongo Fault Line) and of order 100 years after the events along the easter block boundaries (figure 4).

We arranged the one-dimensional stick-slip model to the block rotation model of Southwest Japan studied by Kanaori *et al.* (1991*a*, *b*, 1992, 1993).

We made three circles of stick-slip chain and one pair of straight stick-slip chain to explain the seismic coupling in this area (figure 5). To avoid complication, we enclosed HKFL to West Nankai Trough and also enclosed East Nankai Trough to the boundaries of three rotating blocks.

The simulated earthquake activities are shown in figure 6 which agrees the space

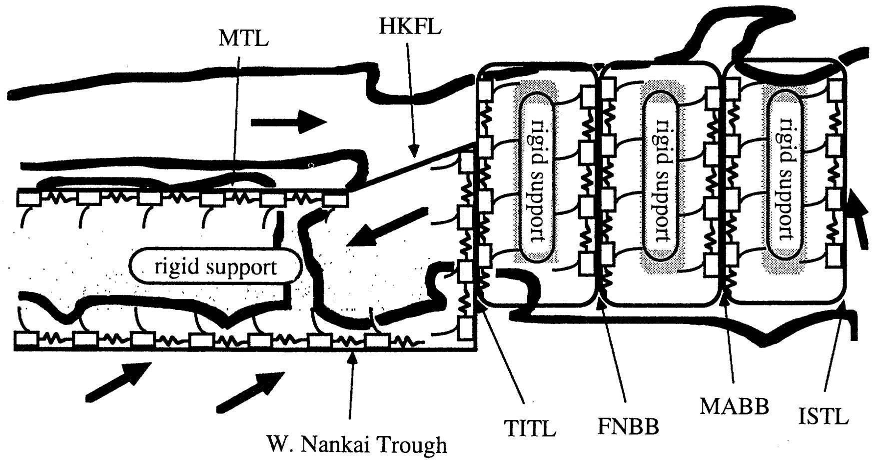

Figure 5. The schematic representation of the caterpillar model. In this model, we make three rotating blocks and one sliding block due to the plate motion. The inside of each block is assumed to be a rigid support.

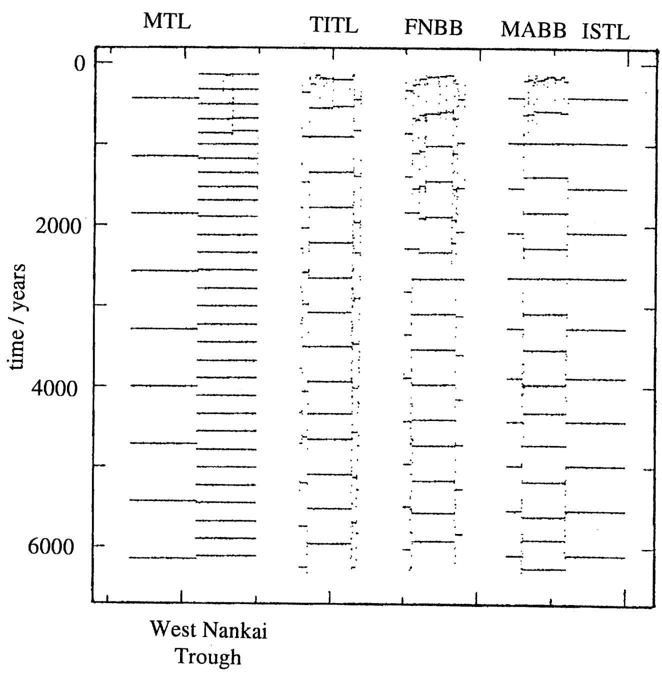

Figure 6. The simulated space-time patterns of the caterpillar model. The length of each line corresponds to the magnitude of each event. We can find the seismic coupling and stable periodicity.

time pattern of real historical destructive earthquakes in this area. Note that the spatial patterns of rupture zones of large events are stable for a long period. This suggests that the block boundary lines can be classified into several segments which generate earthquakes of quite similar sizes in each active period. This observation should be consistent with the characteristic earthquake model. We also calculated the seismic coupling between the block boundaries. The recurrence time of the modelled seismic events at the West Nankai Trough is about 150–180 years, consistent with the observations. According to this caterpillar model, the coupling between these block boundaries is strong and stable for a long time. This result shows us that the predictability of earthquakes is not so small, if we only see the destructive earthquakes over a long time.

The main conclusion of this simulation is that the seismic coupling of large events can be represented by this caterpillar model. The recurrence time of active periods is stable, while the spatial patterns of seismic events along the block boundaries should vary over a long timescale.

The caterpillar model studied here should provide a physical basis to the empirical approaches to destructive earthquake prediction.

This paper owes much to the thoughtful and helpful comments of Professor Kawakami, Professor Kanaori Professor Ito and Professor Takayasu, and I thank them. I also thank the editor and referees for their very useful comments and suggestions.

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